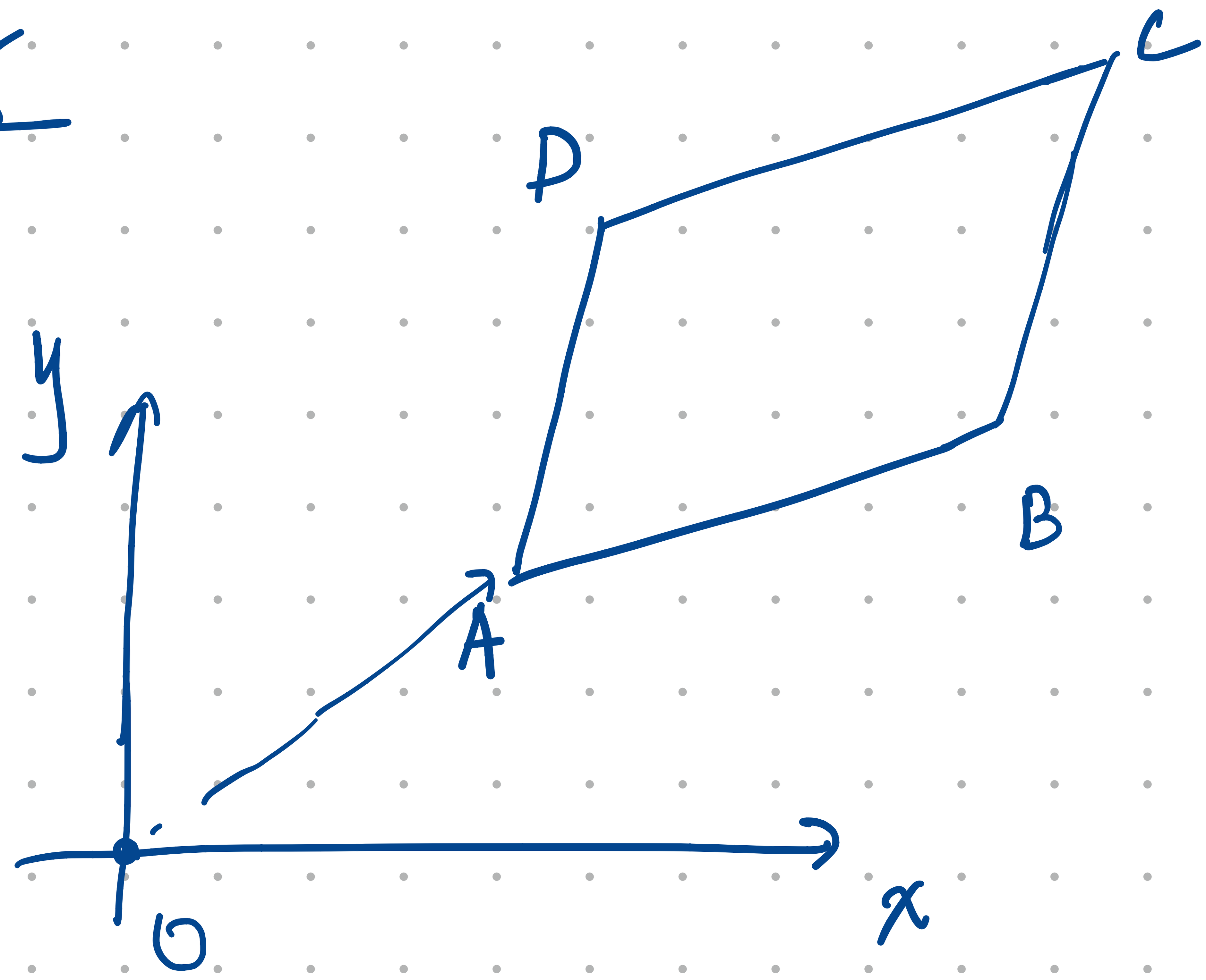
Agenda

- Go over Quiz 8
- HW Questions?
- Examples

Quiz 8

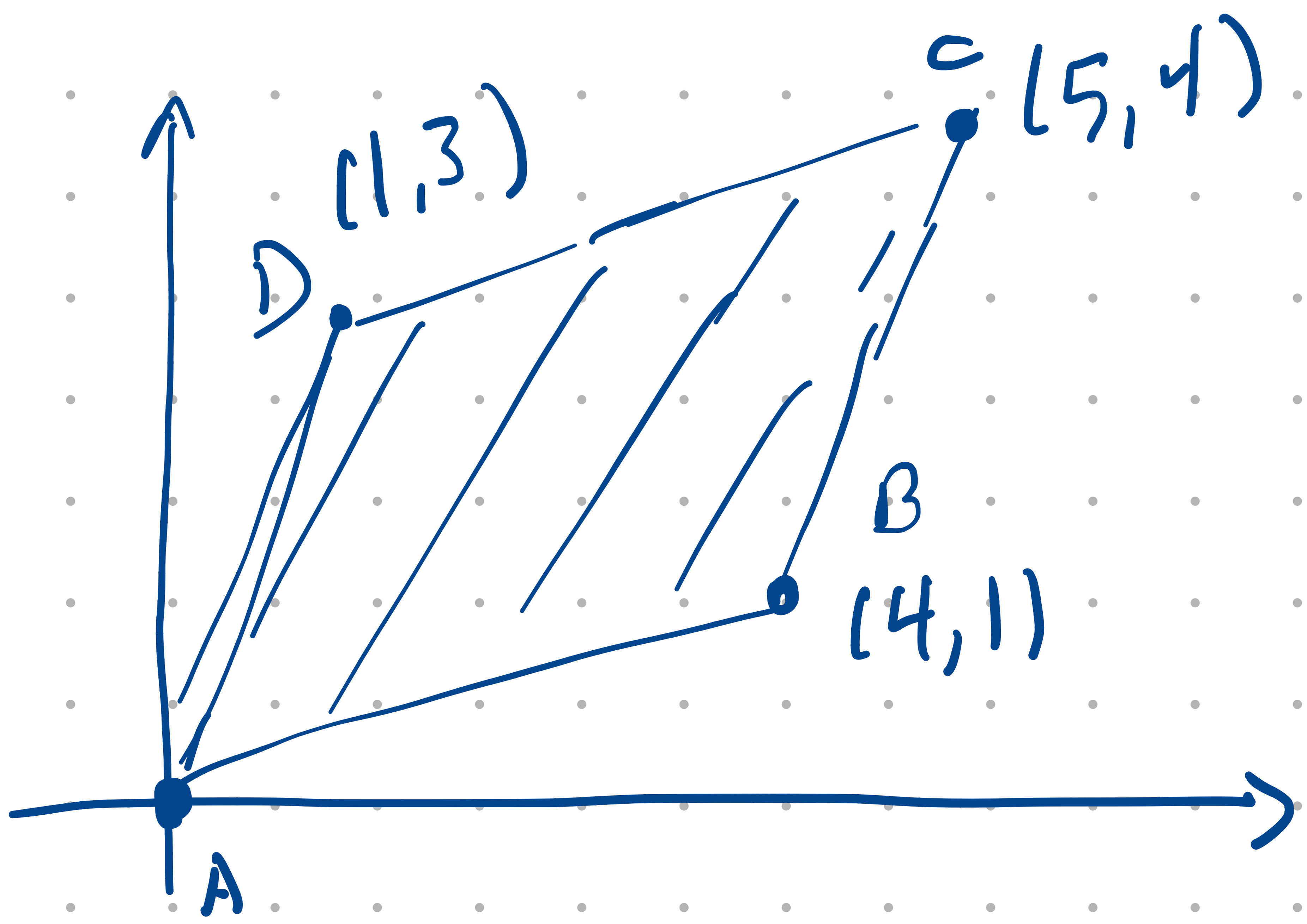
#1)



$$\langle x, y\rangle = \overrightarrow{OA} + u\,\overrightarrow{AB} + v\,\overrightarrow{AD}$$

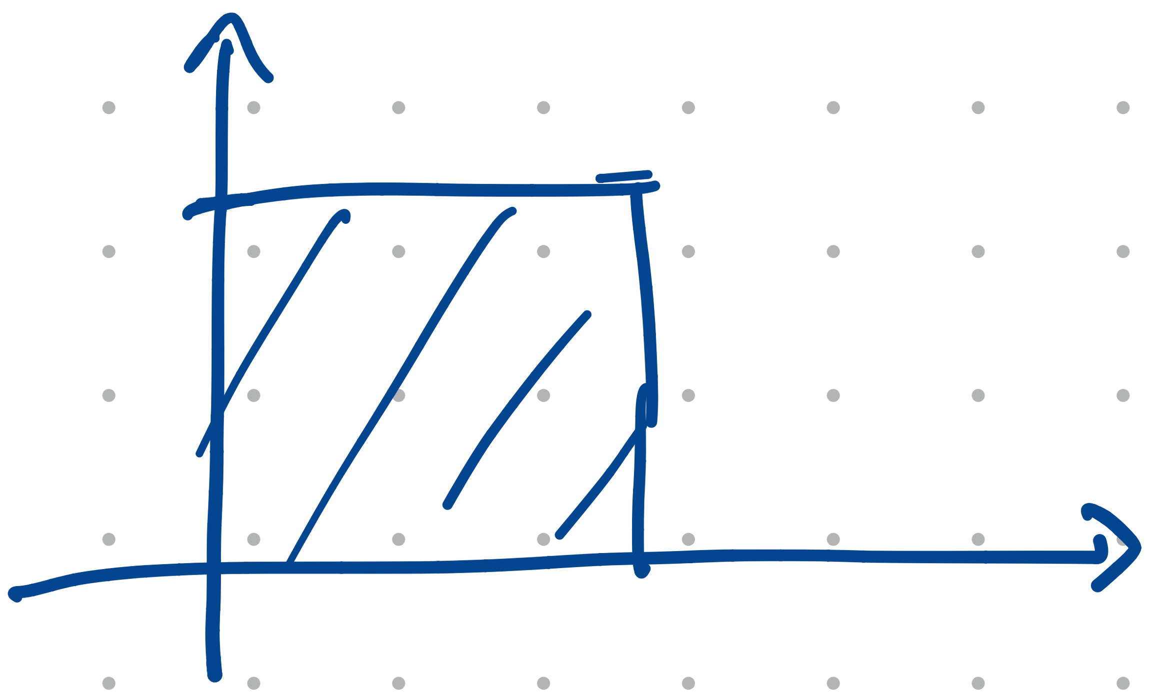
$0 \le u \le 1$

$0 \le v \le 1$

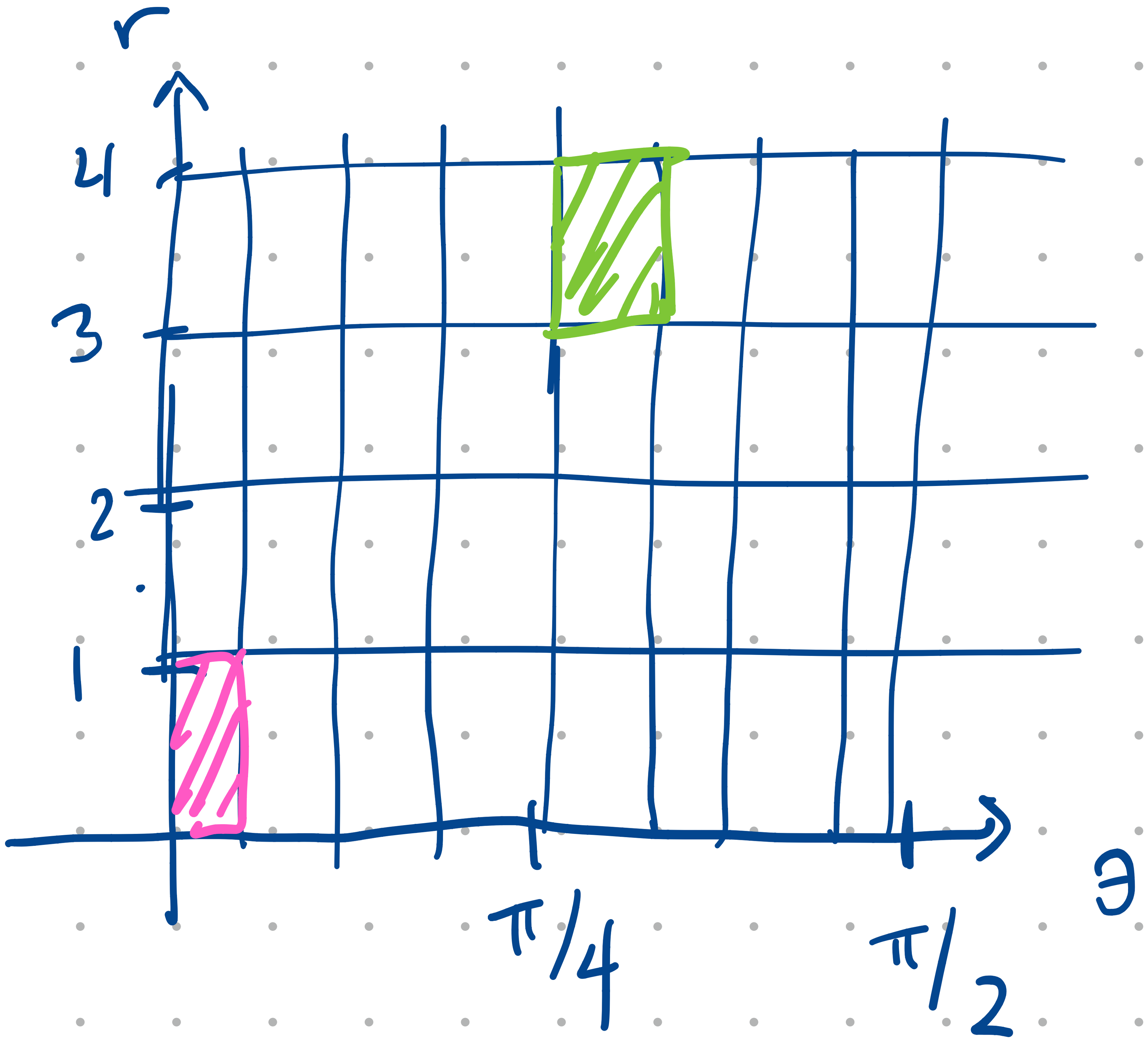


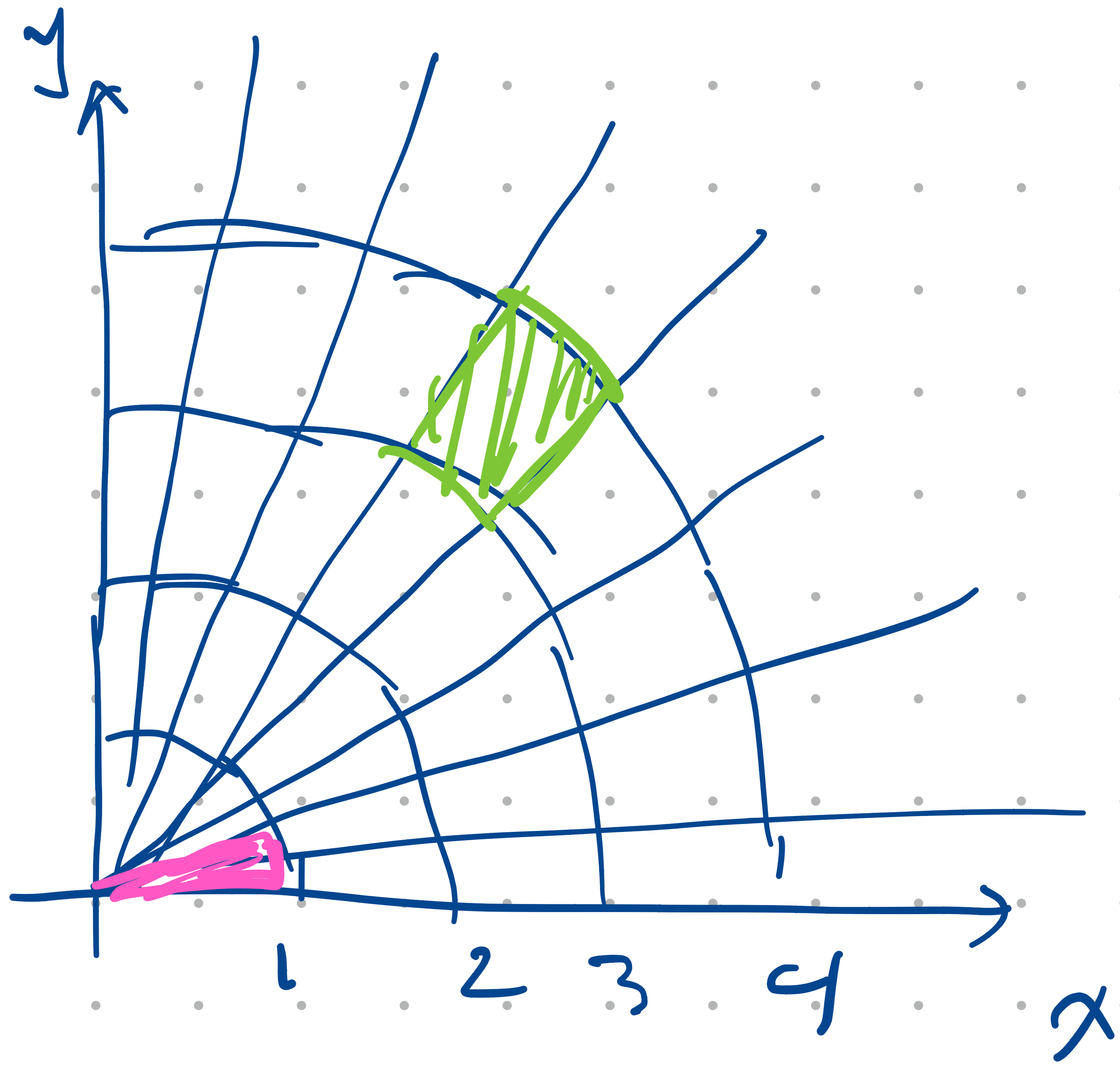
$$\langle x,y\rangle = \langle 0,0\rangle + u\langle 4,1\rangle + v\langle 1,3\rangle$$

$$x = 4u + v$$
$$y = u + 3v$$



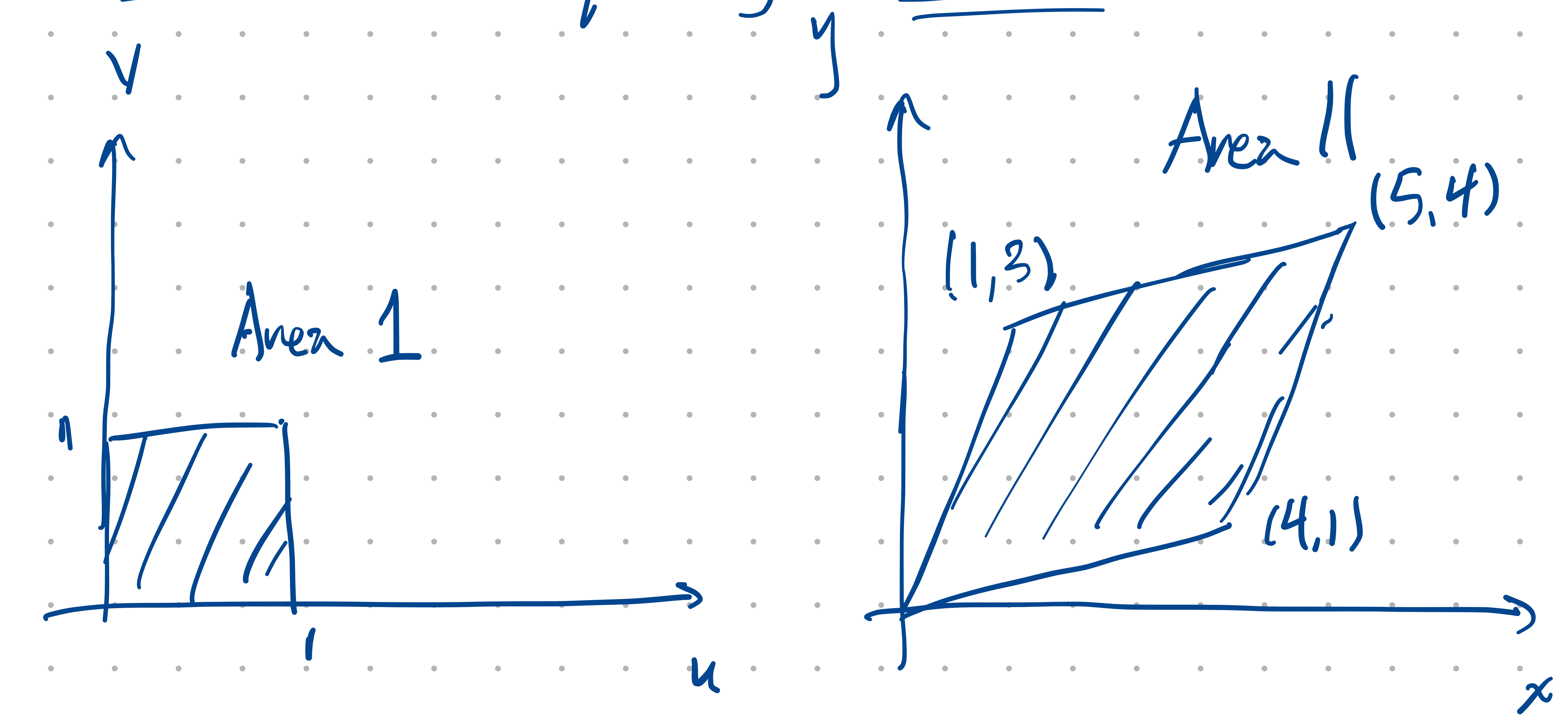
$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \left|\det\begin{bmatrix}4 & 1\\ 1 & 3\end{bmatrix}\right| = |12-1| = 11.$$





The (abs val) of the Jacobian determinant records the ratio of the area of an infinitesimal x,y region to the area of the corresponding infinitesimal u,v region (or in this case, r,θ)

In the quiz problem, the transformation was said to be linear, so this quantity is constant.

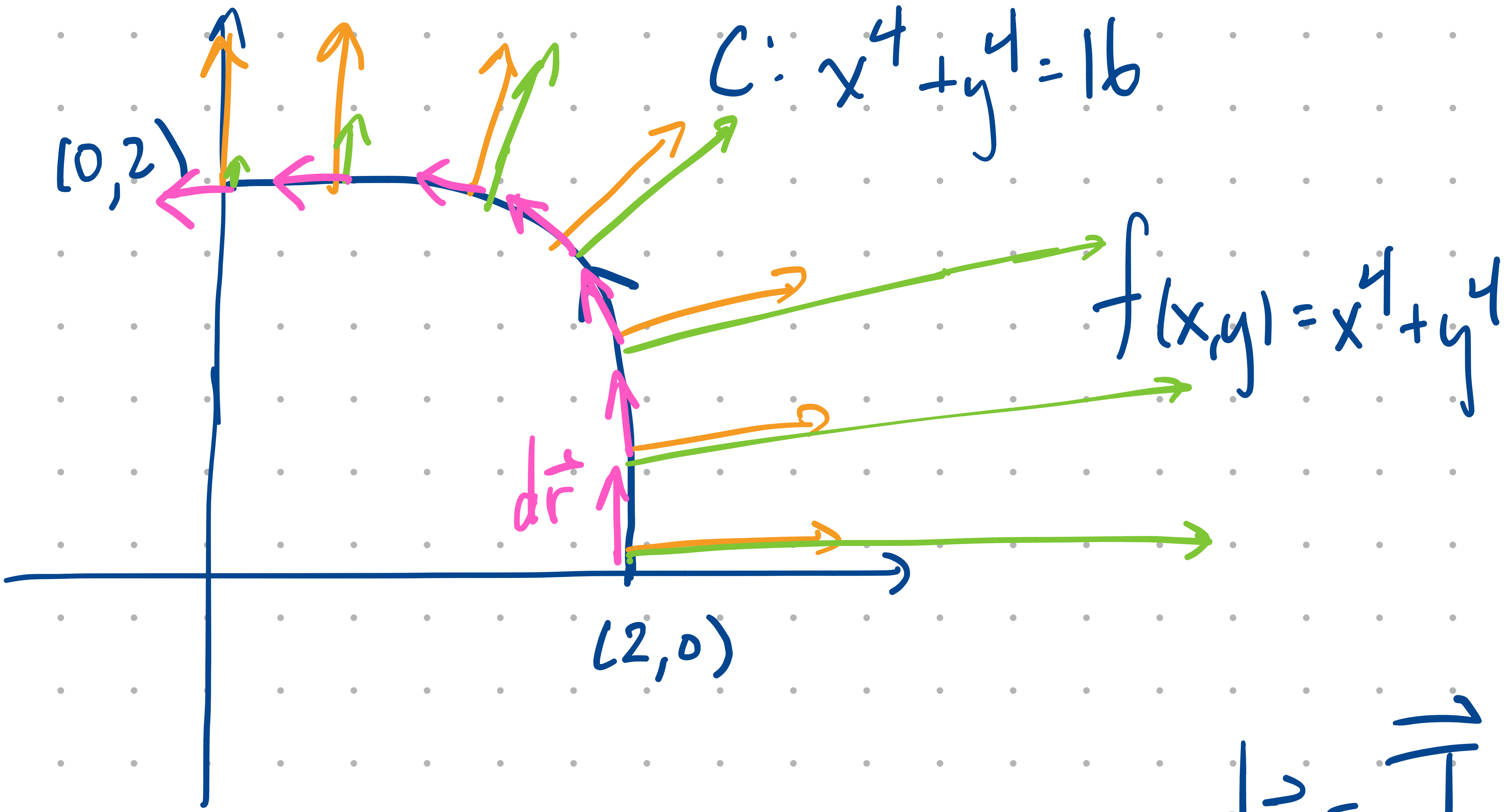


$$\langle 4,1,0\rangle \times \langle 1,3,0\rangle = \langle 0,0,11\rangle$$

So area of parallelogram is 11

So Answer is $11/1 = 11$.

#2)



$$\underbrace{x^2}_{\text{scalar}}\ \underbrace{\nabla f}_{\text{vector}}$$

$$d\vec{r} = \underbrace{\vec{T}}_{\text{unit tangent}}\,ds$$

$$= F'(t)\,dt$$

$$\int_C \underbrace{(x^2\nabla f)\cdot d\vec{r}}_{=0} = \int_C (x^2\nabla f)\cdot \vec{T}\,ds$$

b/c vecs perpendicular

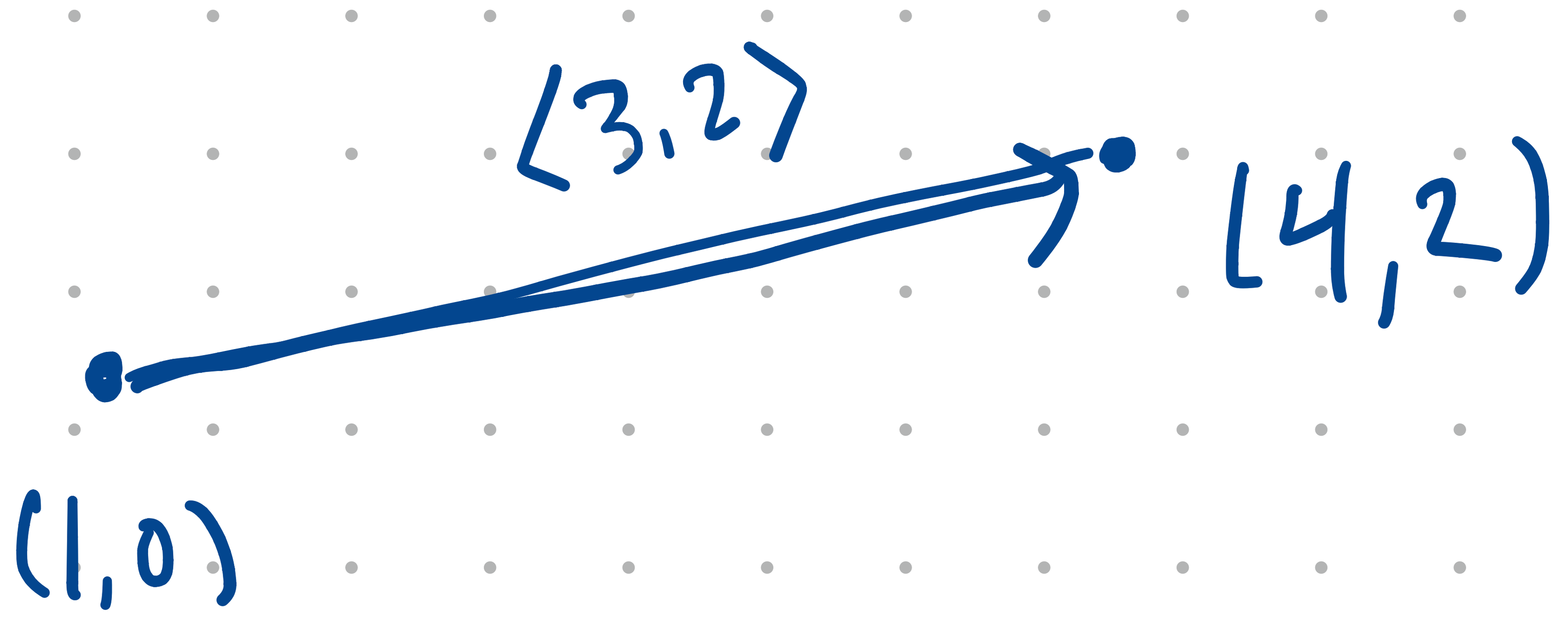
⚠ The gradient is perpendicular to level sets.

	scalar	vector
constant	25	$\langle 7, -3 \rangle$
variable	$x^2 + 2y$	$\langle 2x, \sin(y) \rangle$

$$x^2 \nabla f = x^2 \langle f_x, f_y \rangle$$

$$= \langle x^2 f_x, x^2 f_y \rangle$$

#3) a)



$$\boxed{\begin{aligned} x &= 1+3t \\ y &= 2t \end{aligned}}$$

$$\vec{r}(t) = \langle 1,0\rangle + t\langle 3,2\rangle \qquad 0 \le t \le 1$$

or

$$= (1-t)\langle 1,0\rangle + t\langle 4,2\rangle$$

b) $$\int_C y\,ds = \int_0^1 2t\sqrt{3^2+2^2}\,dt$$

$$= \sqrt{13}\int_0^1 2t\,dt = \boxed{\sqrt{13}}$$

Example

a) Find a potential fn. for

$$\langle (1+xy)e^{xy},\ x^2 e^{xy} \rangle$$

b) Let C be $x^2+y^2=1$, ccw. Compute

$$\int_C (1+xy)e^{xy}\,dx + (x^2 e^{xy} + x)\,dy$$

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a) $f_y(x,y) = x^2 e^{xy}$

$$\boxed{f(x,y) = x e^{xy} + C(x)}$$

$$(1+xy)e^{xy} = \boxed{f_x(x,y) = e^{xy} + xye^{xy} + C'(y)}$$

So $C'(y) = 0$, so

$$f(x,y) = e^{xy} + xye^{xy} + \text{constant}.$$

"Find a potential fn.." ... so we can let the const just be 0.

Note: the integral in b) is $\int_C \left(\vec{F} + \langle 0, x \rangle\right) \cdot d\vec{r}$.

$$= \int_C \vec{F} \cdot d\vec{r} + \int_C \langle 0, x \rangle \cdot d\vec{r}$$

$$= 0 + \int_0^{2\pi} (\cos t)(\cos t)\, dt$$

$$= \cdots = \pi$$

Soon, we will learn an alternative way of doing ⓑ, via <u>Green's Thm</u>.